

Quantum Information with Solid-State Devices

VO I4I.246
Dr. Johannes Majer

Lecture 6



Quantum Information with Solid-state Devices

News

The next class (3. May) will start at the regular time **14:00** at the Raum der Fachgruppe.

Lecture Notes

15 March 2010 - Lecture 1 Introduction

[QISS Lecture Notes 1.pdf](#)

22 March 2010 - Lecture 2

[QISS Lecture 2 Notes.pdf](#)

[QISS Lecture 2 Slides.pdf](#)

[Matlab Lecture 2.zip](#)

12 April 2010 - Lecture 3

[QISS Lecture 3 Notes.pdf](#)

[QISS Lecture 3 Slides.pdf](#)

19 April 2010 - Lecture 4

[QISS Lecture 4 Notes.pdf](#)

[Matlab Lecture 4.zip](#)

26 April 2010 - Lecture 5

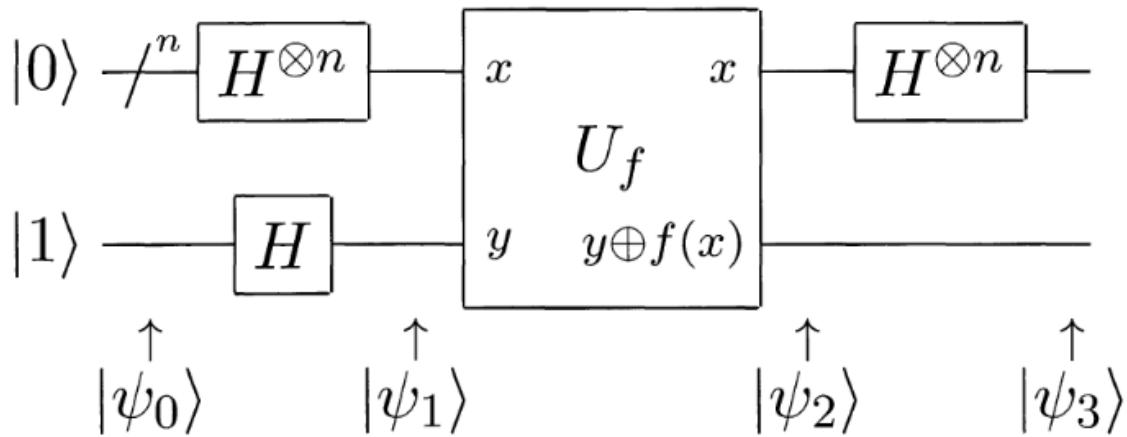
[QISS Lecture 5 Notes.pdf](#)

Lecture Announcement

Lecture 141.246
Quantum Information with Solid-state Devices
Dr. Johannes Majer (Prof. J. Schmiedmayer)



Deutsch-Josza



```
%% Implementation of Uf
%constant function
Ufconst0 = [1 0 0 0 0 0 0 0;...
            0 1 0 0 0 0 0 0;...
            0 0 1 0 0 0 0 0;...
            0 0 0 1 0 0 0 0;...
            0 0 0 0 1 0 0 0;...
            0 0 0 0 0 1 0 0;...
            0 0 0 0 0 0 1 0;...
            0 0 0 0 0 0 0 1];
```

```
%%
psi0 = kron([1 0 0 0],[0 1])
psi0 =
0
1
0
0
0
0
0
0

psi1 = kron(Had2,id)*kron(id2,Had)*psi0
psi1 =
0.3536
-0.3536
0.3536
-0.3536
0.3536
-0.3536
0.3536
-0.3536
```

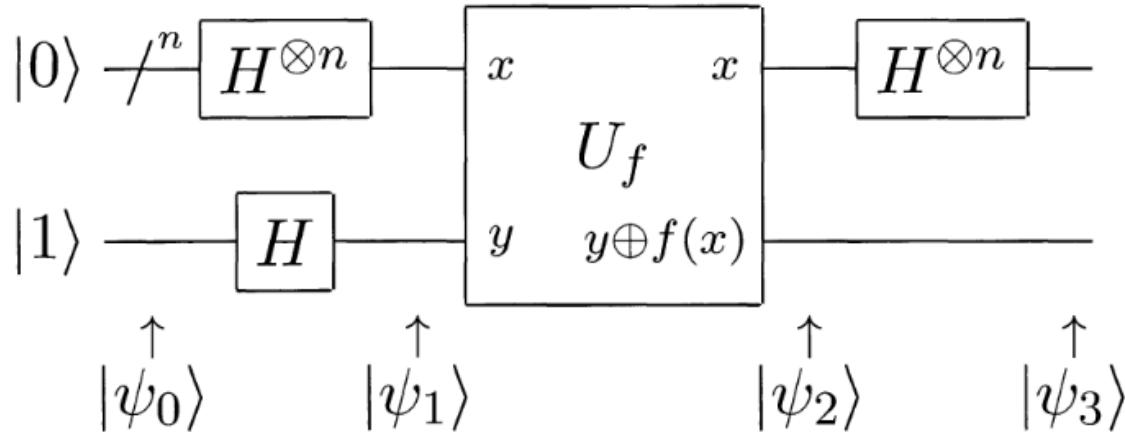
```

psi2 = Uf*psi1
psi2 =
0.3536
-0.3536
0.3536
-0.3536
0.3536
-0.3536
0.3536
-0.3536

psi3 = kron(Had2,id)*psi2
psi3 =
0.7071
-0.7071
0
0
0
0
0
0

sz1 = psi3'*kron(kron(sigma_z,id),id)*psi3
sz1 =
1.0000
sz2 = psi3'*kron(kron(id,sigma_z),id)*psi3
sz2 =
1.0000
```

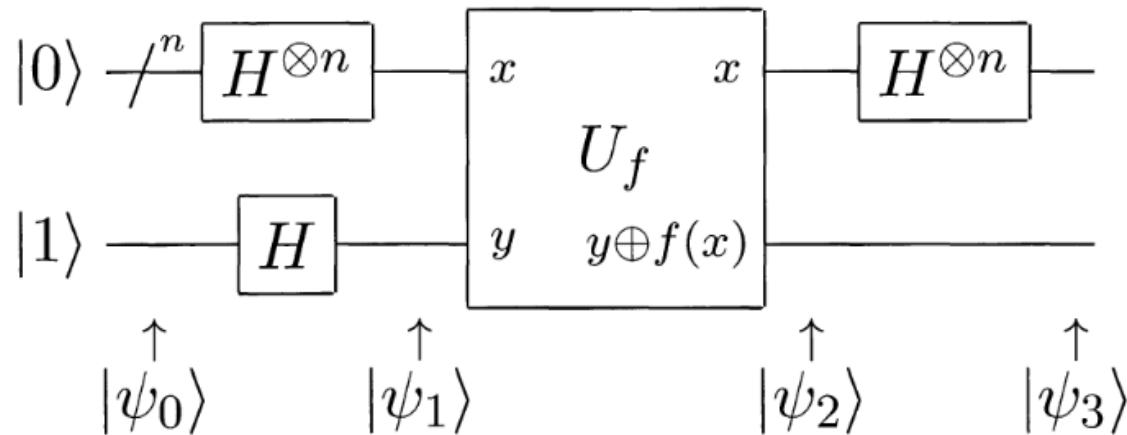
Deutsch-Josza



```
% balanced functions
% f(0,0)=1
% f(0,1)=0
% f(1,0)=0
% f(1,1)=1
Ufb1001 = [0 1 0 0 0 0 0 0;...
             1 0 0 0 0 0 0 0;...
             0 0 1 0 0 0 0 0;...
             0 0 0 1 0 0 0 0;...
             0 0 0 0 1 0 0 0;...
             0 0 0 0 0 1 0 0;...
             0 0 0 0 0 0 0 1;...
             0 0 0 0 0 0 1 0];
```

```
sz1 = psi3'*kron(kron(sigma_z,id),id)*psi3
sz1 =
-1.0000
sz2 = psi3'*kron(kron(id,sigma_z),id)*psi3
sz2 =
-1.0000
```

Deutsch-Josza



```
% f(0,0)=1
% f(0,1)=1
% f(1,0)=0
% f(1,1)=1 Note: this function is neither balanced nor constant
Ufb1101 = [0 1 0 0 0 0 0 0;...
             1 0 0 0 0 0 0 0;...
             0 0 0 1 0 0 0 0;...
             0 0 1 0 0 0 0 0;...
             0 0 0 0 1 0 0 0;...
             0 0 0 0 0 1 0 0;...
             0 0 0 0 0 0 0 1;...
             0 0 0 0 0 0 1 0];
```

```
sz1 = psi3'*kron(kron(sigma_z,id),id)*psi3
sz1 =
-8.3267e-17
sz2 = psi3'*kron(kron(id,sigma_z),id)*psi3
sz2 =
5.5511e-17
```

Quantum Gates

PHYSICAL REVIEW A 67, 032301 (2003)

Natural two-qubit gate for quantum computation using the XY interaction

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(Received 30 July 2002; published 10 March 2003)

CNOT (controlled not) and single qubit rotations are universal

$$\begin{array}{c} \text{CNOT gate symbol} \\ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{pmatrix} . \end{array}$$

$$X = \begin{array}{c} \text{Quantum circuit diagram} \\ \text{with two horizontal lines} \\ \text{and four vertical lines connecting them.} \end{array}$$

$$\begin{pmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{pmatrix}$$

SWAP is not universal
can not create entanglement

Quantum Gates

Unitary Evolution

$$U = \exp(-iHt/\hbar)$$

$$H = E^{ZZ} \sigma_z^{(1)} \otimes \sigma_z^{(2)}$$

Ising interaction

$$\exp\left[-i\mathcal{H}_{i,j}^{ZZ}(E_{i,j}^{ZZ})\frac{\pi}{E_{i,j}^{ZZ}}\right] = e^{i\pi/4} \begin{pmatrix} 1 & & & \\ & -i & & \\ & & -i & \\ & & & 1 \end{pmatrix}$$

$$\overbrace{-\boxed{H}-}^{\text{---}} \overbrace{[\text{-}\pi/2]_Z \begin{pmatrix} 1 & & \\ & i & \\ & & i \\ & & & 1 \end{pmatrix} \overbrace{-\boxed{H}-}^{\text{---}}}^{\text{---}} = \begin{array}{c} \bullet \\ \oplus \end{array}$$

Quantum Gates

XY interaction

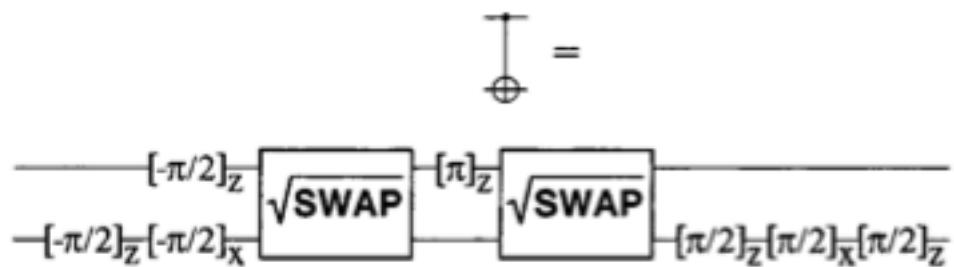
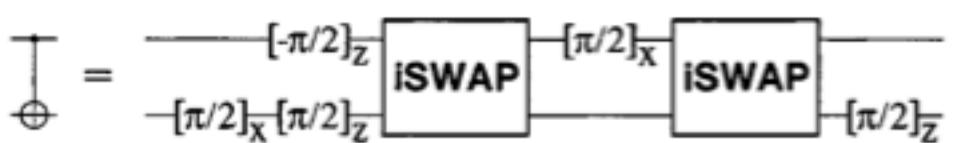
$$\mathcal{H}_{i,j}^{XY}(E_{i,j}^{XY}) = -\frac{E_{i,j}^{XY}}{4} [\sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)}].$$

Heisenberg interaction

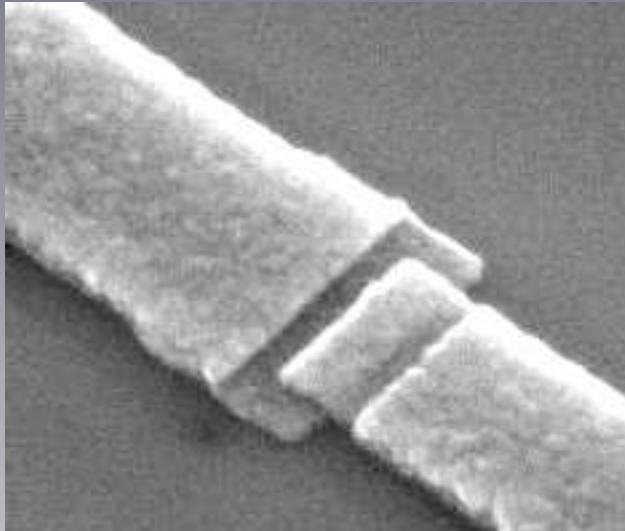
$$\mathcal{H}_{i,j}^{JJ}(E_{i,j}^{JJ}) = -\frac{E_{i,j}^{JJ}}{4} [\sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)} + \sigma_z^{(i)} \sigma_z^{(j)}],$$

$$\boxed{\text{iSWAP}} := \begin{pmatrix} 1 & & & \\ 0 & i & & \\ & & 1 & \\ i & 0 & & \\ & & & 1 \end{pmatrix} = \exp \left[-i \mathcal{H}_{i,j}^{XY}(E_{i,j}^{XY}) \frac{\pi}{E_{i,j}^{XY}} \right].$$

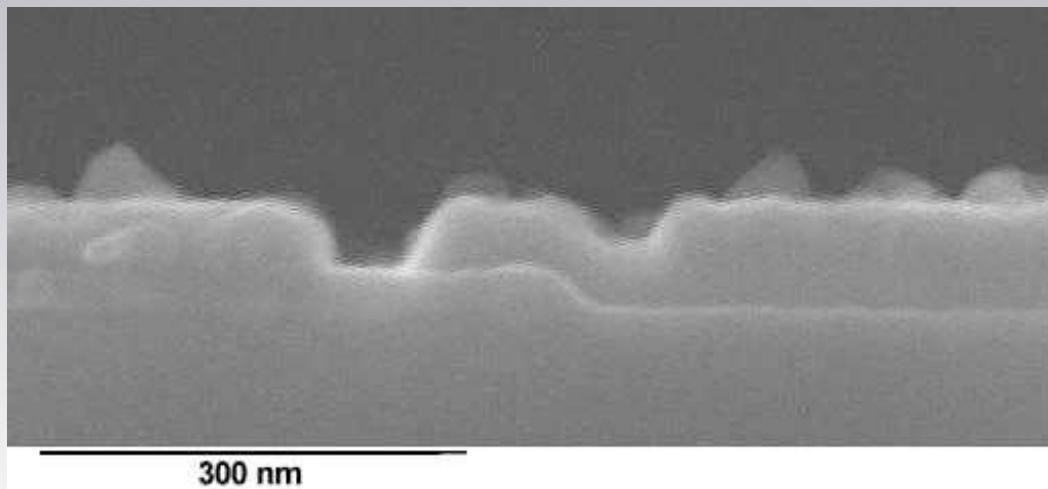
$$\exp \left[-i \mathcal{H}^{\mathcal{IJ}} \frac{\pi}{2E^{JJ}} \right] = e^{i\pi/8} \begin{pmatrix} 1 & & & \\ & \frac{1+i}{2} & \frac{1-i}{2} & \\ & \frac{1-i}{2} & \frac{1+i}{2} & \\ & & & 1 \end{pmatrix}$$



Josephson junction



$$I = I_0 \sin(\delta)$$



$$V = \frac{\phi_0}{2\pi} \dot{\delta} = \frac{\hbar}{2e} \dot{\delta}$$

tilted washboard potential

